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Aizhan Bolatbayeva

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A TWO-COUNTRY MACROECONOMETRIC MODEL

Aizhan Bolatbayeva*

NAC Analytica, Nazarbayev University, Nur-Sultan

Abstract

This paper presents a two-country macroeconomic model for the economies of Kazakhstan and Russia. The model can be used for interpreting the structural relationship between the two economies, determining the degree of trade integration and implementing scenario analyses with various shocks. Single-country models are linked through bilateral trade and exchange rate equations. The baseline simulation of the two-country model demonstrates a good accuracy in tracking the actual dynamics of macroeconomic indicators in both countries. Scenario analyses are conducted with a risk premium shock in the bilateral exchange rate and a monetary policy shock in Russia to analyze the transmission mechanism of the shocks, and clarify on the kind of interdependency of the economies. The model shows a larger influence of the risk premium shock on economic activity in Kazakhstan than in Russia. A two percentage point decline in the key rate does not impose significant inflationary pressure while imports and the real exchange rate are the most affected variables in both countries.

Keyword: Two-country macroeconomic model; Cowles Commission approach; Structural macroeconomic model; Scenario analysis.

JEL code: B22, E17, E27

*Author at: Economic Modeling Development Center, NAC Analytica, Nazarbayev University, 010000 Nur-Sultan, Kazakhstan. Tel.: +7 7172 69 45 16

Email address: aizhan.bolatbayeva@nu.edu.kz (Aizhan Bolatbayeva)

1. Introduction

Multi-country models have become more important with the increasing interdependence of economies and the steadily growing role of economic unions due to the globalization. These models are suitable tools in analyzing bilateral and multilateral economic relations, and evaluating the impact of internal shocks on other economies. The present paper develops a two-country macroeconomic model for the economies of Kazakhstan and Russia. Single-country models possess a similar size and structure. In total, the models contain 107 equations and identities. Both single-country models consist of five blocks: supply, demand, labor market, financial market and government sector. The majority of the equations are built in error correction form, whereas other equations are modeled via ARIMA and Tobit regression. The single-country models are linked to each other through the equations of bilateral trade in goods and services and the bilateral nominal exchange rate. In general, there exists a discrepancy between the data on exports of a shipping country and the data on imports of a recipient country, but we use the data on bilateral exports to estimate the bilateral trade flow. Hence, we specify the bilateral imports in the model as identities. The bilateral nominal exchange rate in the model is defined in line with the uncovered interest parity condition (UIP). The ex-post simulation of the two-country model is conducted to assess its property in tracking the actual dynamics of important endogenous variables. We also assess how various shocks affect the dynamics of model variables. The transmission mechanism of shocks across countries is an issue of immense interest, since Kazakhstan and Russia have been closely tied by trade channels. We discuss the impact of two shocks: a risk premium shock and a monetary policy shock. The responses of endogenous variables are consistent with economic theory, but the impact of the shocks on endogenous variables varies depending on the direction and the magnitude of the shocks.

The two-country model largely builds upon [Weyerstrass \(2015\)](#) who presents a multi-country macroeconomic model for the economies of the former Yu-

goslavia (Slovenia, Serbia, Croatia and Bosnia and Herzegovina). These economies are linked to each other through the equations of bilateral trade and bilateral exchange rate, and he investigates the forecasting accuracy of the multi-country model. The present model also relies on [Dreger and Marcellino \(2007\)](#) who builds a structural macroeconomic model for the Euro area using aggregate quarterly data to generate forecasts and conduct policy analysis for the Euro economy. One of the early works on this topic is the Multicountry Model of the Federal Reserve Board which is described in detail by [Edison et al. \(1987\)](#). The model is presented as a system of five individual quarterly macroeconomic models of the USA, Canada, Germany, Japan and the United Kingdom. The properties of the Federal Reserve model are highlighted by a set of simulation exercises. There exist ample two-country macroeconomic models seeking to examine different issues on the economic interdependence of trading partners. For example, [Fair \(2018\)](#) develops a two-country theoretical model to perform simulation exercises by changing the values of exogenous variables. [Lipton and Sachs \(1983\)](#) present a two-country macroeconomic model with perfect foresight of economic agents to analyze the role of asset prices in terms transmission channels of shocks from one country to another. [Nishimura et al. \(2006\)](#) discusses a two-country model with Cobb-Douglas production function and decreasing returns to scale to investigate how endogenous business cycles of each country transmit to the world through international trade channel. [Rabanal and Tuesta \(2010\)](#) uses a Bayesian approach to explain exchange rate fluctuations in a two-country model for the US and the Euro area economies. [Turnovsky \(1985\)](#) builds a macroeconomic model of two countries with a symmetric behavior to study the impact of monetary and fiscal policies on the economies of interest.

The paper is organized as follows. The next section briefly discusses the data used in the single-country models and the types of adjustments that have been made to the raw data. Section 3 describes the main characteristics and the structure of the two-country models. Section 4 evaluates the ex-post simulation properties of the two-country model. The results of scenario analyses with various shocks are presented in Section 5. The last section provides concluding

remarks.

2. Data description

A data set consists of quarterly variables for Kazakhstan and Russia spanning the period from 2001 to 2018. A large part of the data is retrieved from the statistics agencies of the two countries. Other sources include central bank and finance ministries of both countries, as well as Bloomberg and IMF databases. In the absence of quarterly data, the annual data is transformed into quarterly data. Due to the lack of data on capital stock, we construct a new series of the variable using the perpetual inventory method (PIM).

Data on GDP and its expenditure components have been adjusted for each country to obtain the values at constant prices of the first quarter of 2010.¹ The Statistics Committee of Kazakhstan releases quarterly data on volume index for GDP and its expenditure components. These indexes are used to calculate the real values of national accounts variables at constant prices. The procedure of obtaining the values of the abovementioned variables at 2010 prices for the economy of Kazakhstan are discussed in the research paper by [Abilov et al. \(2018\)](#). The Federal State Statistics Service of Russia publishes data on national accounts in constant prices which have been changed four times since 2001, namely in 2003, 2008, 2011 and 2016. The real values of the variables have been adjusted to 2010 prices using price the deflators as described in [Bolatbayeva et al. \(2020\)](#).

A statistical discrepancy has been detected between the GDP calculated by the production approach and the GDP calculated as the sum of expenditure components of GDP. While both of these approaches should yield an identical GDP figure in theory, they do not produce the same number in practice. This issue has been addressed by adding the statistical discrepancy to inventory investment of gross capital formation in the national accounts.

¹For simplicity, the first quarter of 2010 is referred as 2010 prices

Data on bilateral exports and imports have been obtained in US dollars from the statistics agencies. In theory, exports and imports between trading partners should be consistent in the sense that the value exports from one country to another should be equal to the value of imports in other direction. However, discrepancies often exist in world trade statistics mostly due to the reporting differences. In our case, the statistical discrepancy has varied by 10% on average during the last five years. The present model uses the data on exports of a shipping country as the correct time series, and we use it to estimate the bilateral trade flow. The data are converted into the corresponding home currency series using the bilateral exchange rate over the relevant sample period.

3. The model

In this section, the structure of the two-country model is briefly outlined. Both single-country models embody a similar structure and consist of the following blocks: supply, demand, labor market, prices, financial market and government sector (see [Table 1](#) in Appendix). All variables are tested for stationarity using Augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) statistical tests. For most variables in levels, the tests detect the presence of a unit root. To convert a non-stationary variable into a stationary time series, we take an year-over-year (YoY) measure of these variables in logarithmic form.² The test results indicate that the logarithm of these variables are stationary in YoY. This also allows one to solve the problem with the presence of seasonality in the data.

The two single-country models together contain 107 equations and identities. The majority of the equations are specified in the error correction form to capture the effect of short run dynamics and the deviation from the long run trend. The rest are estimated via ARIMA model and Tobit regression. To confirm the presence of cointegrating relations, the unit roots tests have been

²A YoY measure is defined as $X_t - X_{t-4}$

performed on the residuals from the regression of dependent variables on a set of independent variables with all variables in the regression appearing in levels..

The supply side of the single-country models is built around the potential output equation defined via the Cobb-Douglas production function with constant returns to scale.

$$Y_t = BA_t K_t^\alpha L_t^{1-\alpha} \quad (1)$$

where A_t and B stand for TFP and some normalizing constant. The shares of labor and capital for the model of Kazakhstan are set to 0.56 and 0.44 respectively. These shares have been calculated by the IMF staff based on the firm-level data for Kazakhstan. At the same time, the shares of labor and capital for Russia are equal to 0.6 and 0.4 accordingly. The remaining part of the block includes equations for TFP and NAIRU estimated via ARIMA, and the equation for labor supply.

Goods market of a single-country model consists of the equations for expenditure components of GDP. Private consumption is modelled based on the Keynesian consumption function and the permanent income hypothesis. In the Russian economy, the medium term government bond rate is also used as a determinant of private consumption. The equation for gross fixed capital formation is estimated using the lagged dependent variable, real demand and real interest rate as explanatory variables. Government consumption is treated as exogenous in both models.

One of the main channels that establishes a link between the single-country models is bilateral trade. Total exports are divided into the equations of exports to a trading partner and the exports to the rest of the world. As discussed in Section 2, the data on Kazakhstan's export to Russia and vice versa are collected in US dollars. Then the exports to Russia in domestic currency units have been subtracted from the total exports to obtain the values for exports to the rest of the world. Bilateral exports depend on the lagged values of itself, the exchange rate, world trade index and foreign demand for domestic goods. Exports to the rest of the world is explained by its lagged value and exogenous variables such

as the world trade index and oil price.

As stated in Section 2, the export series of a shipping country has been used as the corresponding import data of a receiving country. Therefore, the bilateral imports in the two-country model are defined in terms of identities. At the same time, imports from the rest of the world are estimated via error correction form. The total imports of each country are equal to the sum of the two endogenous import variables in the model.

There are two key variables that determine the overall condition of labor market: wages and employment. Nominal wages have been modeled in error correction form, and it depends on its own lagged value, unemployment rate and domestic prices. The labor demand equation is built in the form of Tobit regression with censoring left and right end-points at 0 and 0.99 respectively. The lagged dependent variable, real domestic output and real wages are used as explanatory variables.

The block of aggregate prices consists of equations for inflation rate, GDP deflator and other deflators corresponding to expenditure components of GDP. The inflation rate equation in both single-country models is defined as YoY measure of logarithm of CPI. The price level in Kazakhstan is determined by the US price level and domestic output while the Russian inflation rate is explained by the nominal exchange rate and real GDP. Price deflators of expenditure components are estimated in a similar fashion.

Financial markets consist of equations for the real effective exchange rate (REER), the UIP condition and the medium term government bond yields. This block is the second channel that establishes an interrelationship between the single-country models of Kazakhstan and Russia. The REER equation is defined by the bilateral nominal exchange rate and the difference in price levels between the two economies. Real GDP growth is also a determinant of the Russian REER while Kazakhstan's REER is also explained by its own lagged value. The nominal exchange rate between the two countries is defined by the

UIP condition:

$$E_t \left(\frac{S_{t+j}}{S_t} \right) = i_t - i_t^* + \epsilon_t \quad (2)$$

where S_t refers to the exchange rate in terms of domestic currency, i_t is the domestic interest rate, i_t^* is the foreign interest rate and ϵ_t is the risk premium. The dependent variable is expressed in terms of the currency of Kazakhstan - the tenge. An exchange rate in terms the domestic currency of Russia, ruble, is defined as an identity. The last equation in the financial sector is the medium term government bond rate. In both single-country models, the dependent variable is determined by the lagged dependent variable and a central bank policy rate.

The government sector comprises the equations for the central bank policy rate and the government revenue components. The central bank policy rate equation takes the form of Taylor rule. Thus, in the Russian model, the conduct of monetary policy is affected by its lagged dependent value, the quarter-over-quarter (QoQ) change in the inflation rate and output gap. In a similar vein, the central bank policy rate is explained in Kazakhstan's model, except for the output gap which has been dropped from the equation due to its insignificant coefficient. An exchange rate variable has been included in both equations as an important explanatory variable that might partially affect the decisions of policymakers in resource-oriented countries. In both single-country models, the government expenditure is divided into the following components: personal income taxes, corporate income taxes, VAT revenues, excise taxes and other taxes. The equations for each component are estimated in error correction form.

4. Simulation

This section describes the properties of the ex-post simulation conducted in the two-country model with the results reported in [Figure 1](#) and [Figure 2](#). The simulation of the model runs for the period starting from 2004 to 2018. The

figures illustrate the dynamics of actual and fitted values of relevant macroeconomic variables, including real GDP growth rate, potential output, inflation and unemployment rates.

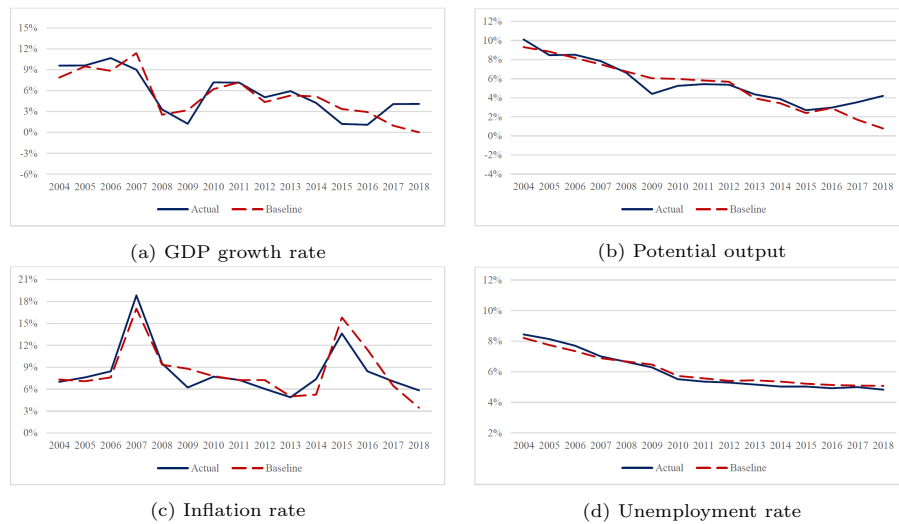


Figure 1: Macroeconomic indicators of Kazakhstan

Figure 1 demonstrates the performance of Kazakhstan’s economy in the baseline simulation. A moderate difference can be observed between actual observations and baseline values of real GDP and potential output growth rates in 2018 (see Figure 1a and Figure 1b). However, there is only preliminary data for 2018 at the time of writing this paper. This might have partially caused the gap between actual and baseline values of the variables. Nevertheless, the two-country model shows a better fit of Kazakhstan’s economy than the single-country model does in the work [Abilov et al. \(2018\)](#).

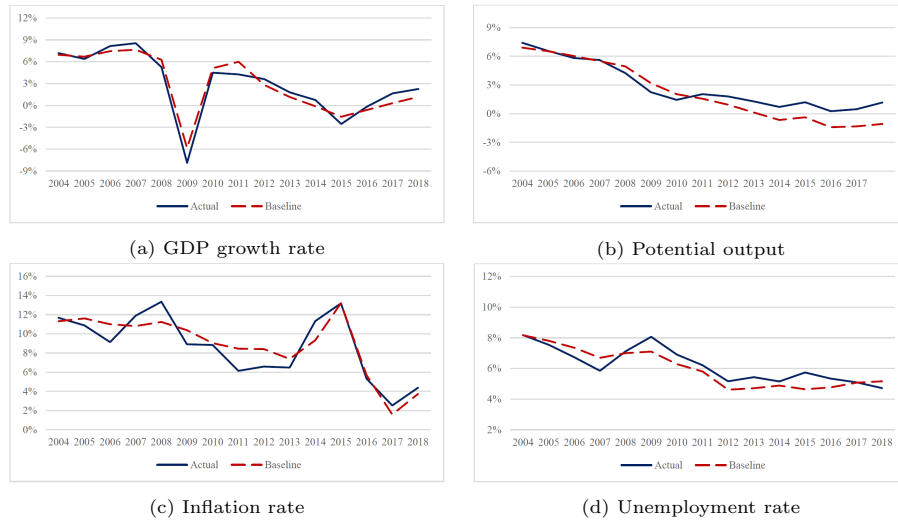


Figure 2: Macroeconomic indicators of Russia

The ex-post simulation results for Russia also show a good accuracy in reproducing the historical growth path of the main endogenous variables (see [Figure 2](#)). The values of the GDP growth rate obtained from the baseline simulation closely match the reported data. Although the ex-post simulation slightly underestimates the baseline values of the growth rate of potential output starting from 2011 (see [Figure 2b](#)), the model tracks accurately the actual dynamics of the endogenous variables.

5. Scenario analysis

The two-country model can be employed to examine possible effects of multiple shocks on the economies and evaluate the transmission mechanism embedded in the model. Two scenario analyses are conducted to investigate these properties of the model. We assess the consequences of a risk premium shock in the bilateral exchange rate and of a monetary policy shock occurring in Russia. The results are expressed in percentage point difference from the baseline simulation.

In the first alternative scenario, we consider a shock to the risk premium in the UIP condition presented in [Equation 2](#). The exchange rate in the UIP

equation is expressed in terms of the domestic currency of Kazakhstan. The risk premium is assumed to increase by 10% in the first quarter of 2011. The impact of the occurred shock on important macroeconomic variables of Kazakhstan and Russia is illustrated in [Figure 3](#) and [Figure 4](#) respectively.

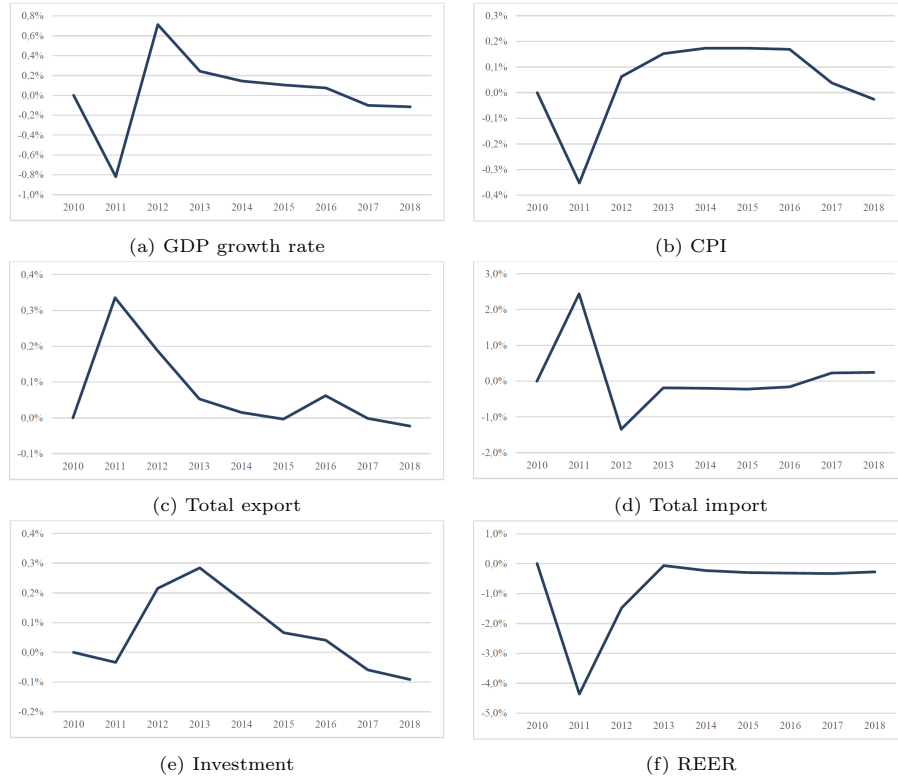


Figure 3: Response of main variables of Kazakhstan to the risk premium shock

The 10% increase in the risk premium triggers the tenge to depreciate. Initially, the GDP of Kazakhstan responds negatively to the shock due to the large but short increase in imports from Russia ([Figure 3d](#)). The latter occurs because the value effect dominates over the volume effect. At the same time, the shock has limited and positive impact on exports of goods and services to Russia. Nevertheless, it cannot offset the growth in imports. According to the Statistics Committee of Kazakhstan, in 2011, Russia accounted for around 42% of total imports and 8.2% of total exports in Kazakhstan. It is thus not sur-

prising that imports have a greater and persistent impact on GDP. After one year, the GDP growth rate accelerates and even outperforms the baseline level moderately (see [Figure 3a](#)). This is mainly due to the fall in imports from the rest of the world stimulated by the lag effect of the real effective exchange rate depreciation. Thus, the trade balance of Kazakhstan recovers in accordance with the J-curve. The response of the inflation rate is similar to the response of the real GDP growth rate.

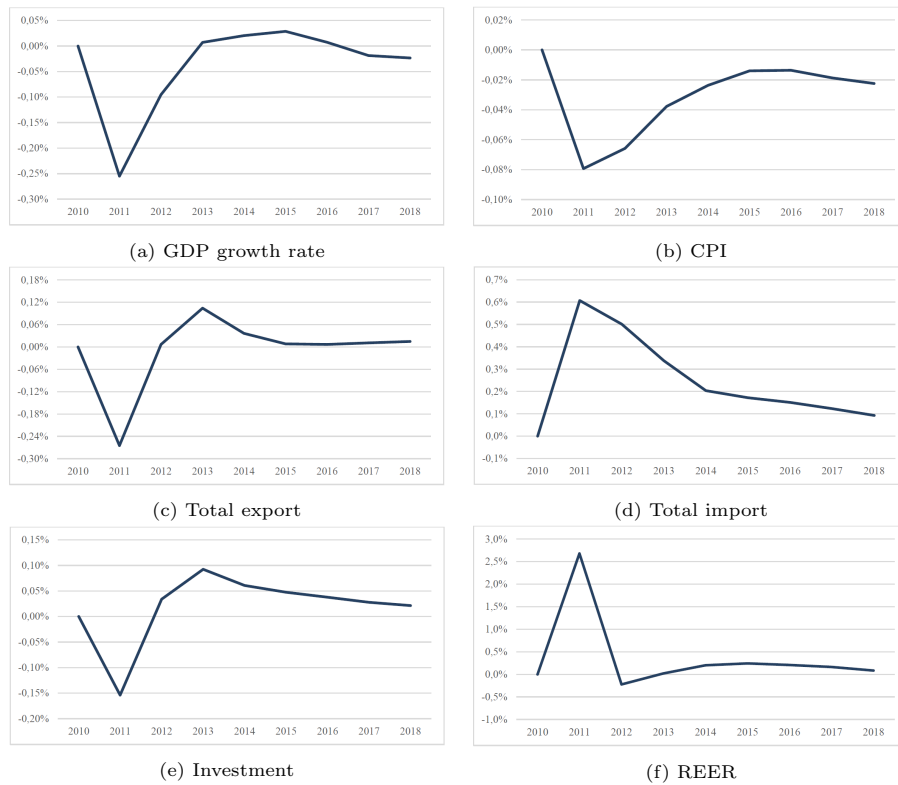


Figure 4: Response of main variables of Russia to the risk premium shock

The impact of the risk premium shock on the Russian economy is smaller than for Kazakhstan. The initial appreciation of the Russian ruble causes an immediate decline in total exports and a surge in total imports (see [Figure 5c](#) and [Figure 5d](#)). These two effects together generate downward pressure on domestic GDP and price levels as illustrated in [Figure 5a](#) and [Figure 5b](#). The

impact of the shock on the variables is temporary. The GDP growth rate and total exports immediately return back to their normal levels, while it takes longer for the inflation rate and total imports to return to the baseline level.

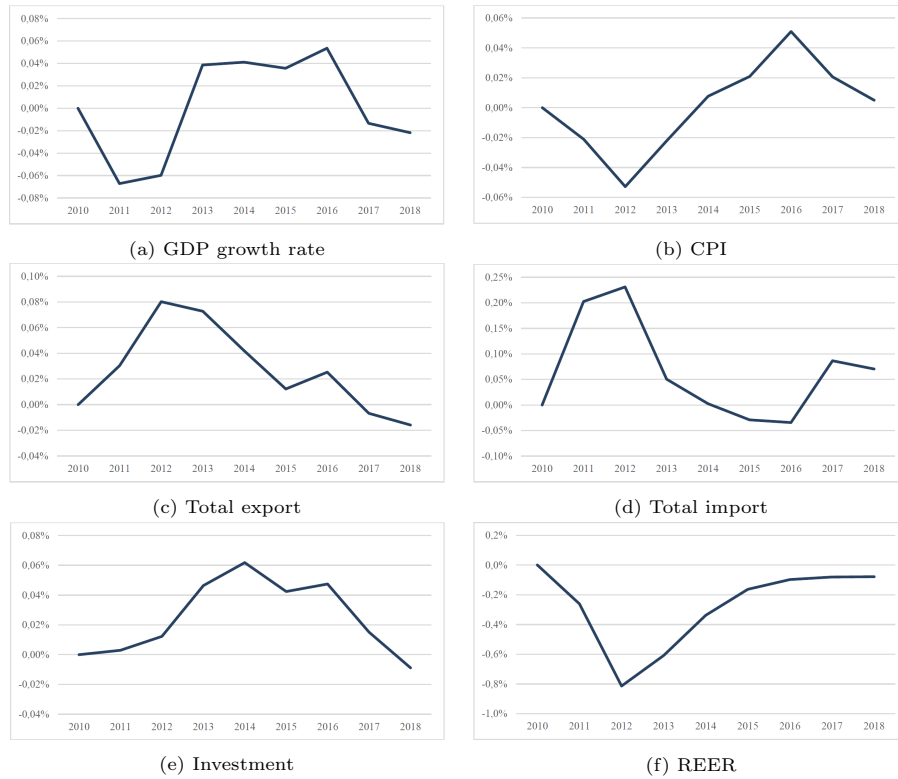


Figure 5: Response of main variables of Kazakhstan to the key rate shock

The second scenario analysis is a two percentage point decline in the central bank policy rate of Russia. The cut in the key rate of Russia leads to an increase in the interest rate differential presented in Equation 2. According to the UIP condition, the increase in the interest rate differential leads to a depreciation of Kazakhstan's currency in the next period. The depreciation of the tenge and the real demand growth in Russia stimulates exports of Kazakhstan as presented in Figure 5c. At the same time, the domestic currency depreciation initially creates the value effect which causes an immediate and short term growth in total imports (see Figure 5d). The shock has imposed greater impact on total

imports than on total exports. As a result, the GDP growth rate and CPI in Kazakhstan fall negligibly recovering very fast afterwards.

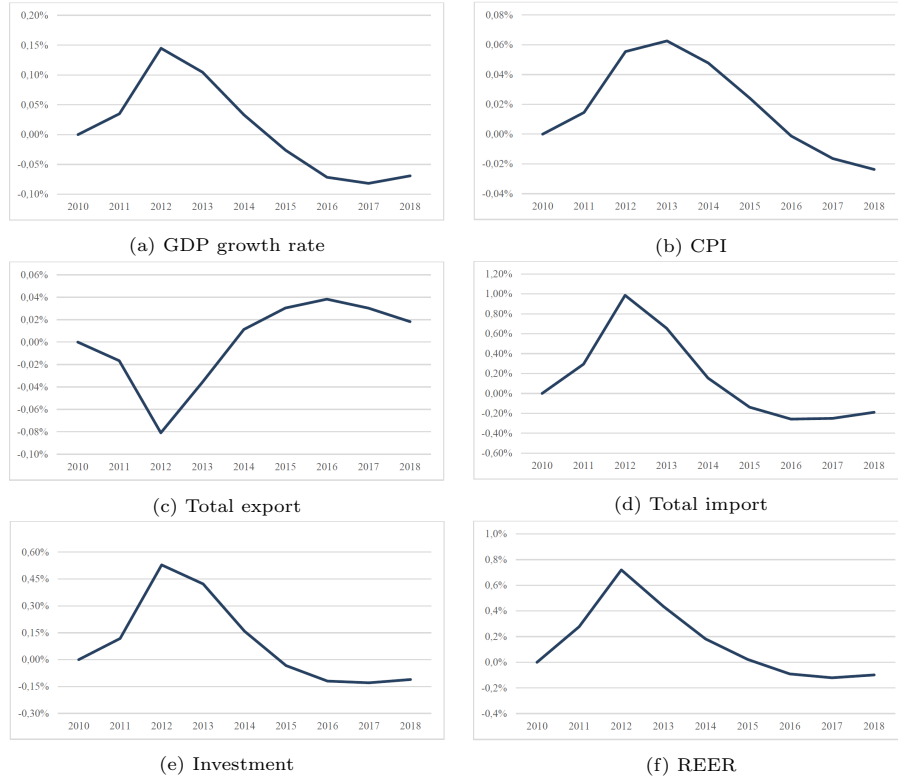


Figure 6: Response of main variables of Russia to the key rate shock

The lower interest rate leads to the appreciation of the domestic currency in Russia. Total exports tend to diminish relative to the baseline scenario resulting from the appreciation of the Russian ruble and the fall in foreign output (see [Figure 6d](#)). At the same time, a surge in the domestic real demand and currency appreciation boost total imports. The maximum deviation of the variable from the baseline scenario attains one percentage point in the following year after the shock. Nevertheless, these effects are offset by the growth in gross fixed capital formation and private consumption which are stimulated by the fall in medium government bond rate. Thus, the GDP growth rate and price levels respond positively to the shock (see [Figure 6a](#) and [Figure 6b](#)). Although the impact of

the shock on most variables of the Russian economy is negligible in magnitude, it lasts for more than five years.

6. Conclusion

This paper presents a two-country macroeconomic model for the economies of Kazakhstan and Russia. The model is built to shed more light on the bilateral relationship between the two economies. Single-models are linked through the equations of bilateral trade and the bilateral nominal exchange rate. The properties of the baseline simulation are examined for the period starting from 2004 to 2018. The simulation results show an excellent performance of the model in terms of tracking the actual dynamics of endogenous variables for both economies. Furthermore, the two-country model simulation fits the data on Kazakhstan's economy better than the single-country model in [Abilov et al. \(2018\)](#).

The integrated model also aims at assessing the responses of endogenous variables to domestic and foreign shocks, as well as analyze the transmission mechanism of the shocks across countries. We have run two simulations to analyze the performance of the model under two shocks. The first simulation examines the influence of a risk premium shock in the bilateral nominal exchange rate. The results indicate that the Kazakhstani economy is more influenced by the shock than the Russian economy. The second scenario analyzes the impact of an increase in the key rate of Russia by two percentage points. The shock does not produce significant pressure on the domestic price level, while it leads to the appreciation of the real exchange rate in Russia, and temporarily increases total imports in both countries. Overall, the two-country macroeconomic model turns out to be a useful tool for analysis as it takes into account all structural linkages between the economies of Kazakhstan and Russia, and also allows us to examine the transmission channels of various shocks and their effects on both economies.

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Appendix

Kazakhstan			Russia	
Exogenous	Endogenous		Endogenous	Endogenous
Working age population	Potential output Labor force NAIRU TFP	Supply	Potential output Labor force NAIRU TFP	Working age population
Oil price World trade	Private consumption Investment Export to other countries Export to Russia Import from Russia	Demand	Private consumption Investment Export to other countries Export to Kazakhstan Import from Kazakhstan	Oil price World trade
	Nominal wages Employment	Labor market	Nominal wages Employment	
Federal funds rate Nominal KZTEUR Nominal KZTUSD	Real effective exchange rate UIP condition KZTRUB Medium term government bond rate	Financial market	Real effective exchange rate Medium term government bond rate	Federal funds rate Nominal RUBEUR Nominal RUBUSD
Personal income tax rate VAT rate	National Bank policy rate Personal income taxes Corporate income taxes VAT revenues Excise taxes Other taxes	Government sector	Central Bank policy rate Personal income taxes Corporate income taxes VAT revenues Excise taxes Other taxes	Corporate tax rate VAT rate

Table 1: The model block