The Importance of Terms of Trade Shocks: Kazakhstan

Alisher Tolepbergen

Document type
Working Paper No.4

This version is available at
http://nacanalytica.com/images/macro/Papers/Terms_of_Trade_Shocks_Kazakhstan.pdf

Citation details
Tolepbergen, Alisher. 2020. The Importance of Terms of Trade Shocks: Kazakhstan. NAC Analytica Working Paper. No.4.

THE IMPORTANCE OF TERMS OF TRADE SHOCKS: KAZAKHSTAN

Alisher Tolepbergen*

NAC Analytica, Nazarbayev University, Nur-Sultan

Abstract

The conventional wisdom assumes that terms of trade shocks are the main drivers of business cycle dynamics in emerging exporting economies. This paper studies the effect of terms of trade shocks on key macroeconomic variables for the Kazakhstani economy. Empirical SVAR model estimates suggest that the terms of trade shocks account for 12 % of output variation for the economy. Further, three sectoral DSGE model with estimated structural parameters predict the modest importance of the terms of trade shocks for small open economy.

Keyword: Terms of Trade; DSGE; SVAR; Kazakhstan.

JEL code: B22, C32, C61,E17, E32, F41.

*Corresponding author at: Economic Modeling Development Center, NAC Analytica, Nazarbayev University, 010000 Nur-Sultan, Kazakhstan. Tel.: +7 7172 69 47 95

Email address: atolepbergen@nu.edu.kz (Alisher Tolepbergen)

Introduction

Terms of trade shocks are viewed as the major source of business cycle fluctuations in small open economies. Particularly, countries exporting primary commodities, such as oil and gas are assumed to be highly dependent on the terms of trade shocks. This paper attempts to study the business cycle dynamics of Kazakhstani economy for the period between 2001 and 2018. Particularly, the paper analyzes the terms of trade shocks and its effect on output fluctuations. The paper estimates SVAR model based on quarterly data and argues that the terms of trade shocks do not account for most of the variance in output. The results of the SVAR estimation show that around 12 % of output variance can be attributed to the terms of trade shocks. Further, the paper adopts the DSGE model, developed by Schmitt-Grohe and Uribe (2018), to apply for the study of business cycle dynamics in Kazakhstan. The theoretical model's predictions regarding the effect of the terms of trade shocks are in line with the SVAR model predictions. Hence, the paper argues that, despite the traditional wisdom, the terms of trade shocks are not the major source of business cycle fluctuations in Kazakhstan.

The paper is structured in the following way. Section 1 provides a literature review on the effect of terms of trade shocks on key macroeconomic variables in emerging markets. Section 2 discusses the data used and presents key empirical regularities for Kazakhstani economy. Section 3 builds an empirical SVAR model to analyze the effects of the terms of trade shocks. Further, the structure of the medium-scale DSGE model with three sectors is discussed in Section 4. Section 5 explains the calibration and estimation of the model. Section 6 compares the empirical SVAR model predictions on the variance of key variables explained with the predictions of the DSGE model. Finally, Section 7 concludes the paper summarizing main results.

1. Literature review

Mendoza (1995) provides one of the first attempts to analyze theoretically the relationship between terms of trade shocks and business cycles for developing economies. The author studies annual empirical regularities for the period between 1955 and 1990 and builds a three-sector DSGE model for 7 advanced economies and 23 developing countries. Mendoza (1995) finds that terms of trade shocks play an important role in business cycles. For emerging economies terms of trade shocks explain about 37% of the variability in GDP.

Drechsel and Tenreyro (2018) use the annual data on Argentina from 1900 to 2015 to study the business cycle dynamics for an exporting emerging economy. Particularly, the authors analyze the effect of commodity price shocks on key economic variables. On the empirical side, the authors estimate SVAR model with commodity prices taken as exogenous. On the theoretical side, they build a medium scale DGSE model for a small open economy. There are two kinds of producers in the economy: a commodity producer and a final good producer. Drechsel and Tenreyro (2018) find that commodity price shocks account for 38% of the variation in output, 42% variation in consumption, and 61% of variation in investment.

Another research investigating the effect of terms of trade shocks on key macroeconomic variables is made by Schmitt-Grohe and Uribe (2018). The authors estimate SVAR model for 38 emerging economies for the period between 1980 and 2011. Terms of trade and interest rate spread are taken to be exogenous. The estimated country-specific SVAR model implies that terms of trade shocks account for only 10 % of variation in output. Further, the authors compare on a country level the predictions of the SVAR model with the results of a theoretical model. The theoretical model in the paper consists of three sectors, with production, domestic demand, capital, and labor in each sector. In addition, the empirical SVAR and the theoretical model share the same exogenous processes. The comparison of the empirical and theoretical models shows that terms of trade shocks do not represent a major source of fluctuations in output for small open economies. Both models predict less than 10 % of variation in the output due to the terms of trade shocks.

2. Data and empirical regularities

This section discusses the data employed in the model and presents the main empirical features of Kazakhstani business cycle from 2001 to 2018. Unlike recent papers on the developing markets business cycles, this paper uses quarterly data due to the lack of reliable and sufficient time series on an annual basis. Hence, the analysis includes 72 observations in total. All the data except for terms of trade and interest rate spread are taken from the Statistics Committee of the Ministry of National Economy of the Republic of Kazakhstan and the National Bank of Kazakhstan. The data on commodity terms of trade are taken from the IMF database. In the construction of the terms of trade index the IMF weights individual commodities by ratio of net exports to GDP on a rolling basis. Figure 1 depicts the quarterly data on the terms of trade for Kazakhstan and crude oil price in USD. It can be seen that the terms of trade highly matches the oil price data. In addition, the correlation coefficient between these two series constitutes around 0.96, which is nearly one-to-one relation. High similarity of Kazakhstani terms of trade to oil price data can be explained by the fact that, on average, 60 percent of total exports are exports of crude oil. Therefore, one can refer to the terms of trade shocks for Kazakhstani economy as oil price shocks as well. Further, Figure 1 shows that the terms of trade experienced two periods of significant drop, one was in 2008 and the other in 2015, respectively. In both periods the terms of trade declined by around 25 percent.

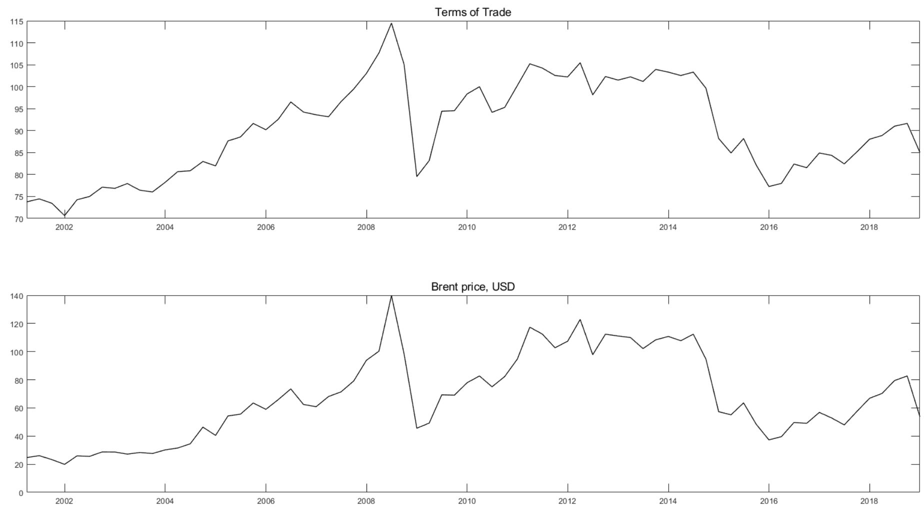

Figure 1: Terms of trade and oil price

The paper follows Akinci (2013) in choosing Baa corporate bond spread, which is defined as the difference between Moody's Baa corporate bond yield and the federal funds rate as the relevant measure of interest spread for developing economies. Other variables used in the empirical model include GDP, private consumption, investment, the trade balance and the real exchange rate. The real exchange rate represents an index of the real exchange rate of Kazakhstani tenge against US dollar. All variables are seasonally adjusted and detrended using the Hodrick-Prescot filter. The trade balance is taken as a share of trend GDP and then detrended. Table 1 below summarizes the key business cycle moments for Kazakhstani economy. The table reports standard deviation, first order autocorrelation and contemporaneous cross-correlation of the variables.

According to Table 1, most of the data moments for Kazakhstani economy are in line with stylized facts of business cycles for developing economies. First, consumption and investment are more volatile than GDP. Second, the terms of trade positively correlates with GDP. Third, there is a negative correlation between interest spread and GDP. Finally, the trade balance negatively correlates

	ToT	S	GDP	C	I	TB	RER
Standard deviation	0.06	0.01	0.027	0.05	0.092	0.01	0.08
Autocorrelation	0.68	0.85	0.54	0.41	0.11	0.46	0.82
Correlation with GDP	0.52	-0.47	1	0.51	0.43	-0.36	-0.35
Correlation with ToT	1	-0.56	0.52	0.36	0.19	-0.07	-0.41
Correlation with S	-0.56	1	-0.47	-0.35	-0.38	0.45	0.18

Table 1: Summary statistics

with output. Unlike most literature on small open economies, this paper finds negative correlation between the terms of trade and the trade balance. However, the coefficient appears to be insignificant.

3. Empirical model

The paper estimates SVAR model with seven aforementioned variables in order to find an empirical measure of the contribution of the terms of trade shocks to business cycles in Kazakhstan. The model takes the form

$$\mathbf{A}_0 X_t = \mathbf{A}_1 X_{t-1} + \mu_t \tag{1}$$

where X_t is a vector of endogenous variables given by

$$X_t \equiv \begin{bmatrix} ToT_t \\ S_t \\ TB_t \\ GDP_t \\ C_t \\ I_t \\ RER_t \end{bmatrix}$$

The variables in X_t are cyclical components of the terms of trade, interest rate spread, the trade balance, output, consumption, investment and the real exchange rate. $\mathbf{A}_0$ and $\mathbf{A}_1$ are 7x7 matrices of coefficients, where $\mathbf{A}_0$ is a lower unit triangular matrix with ones on the main diagonal. μ_t is a 7x1 random vector with mean zero and diagonal variance-covariance matrix Σ. Assuming that $\mathbf{A}_0$ is invertible and multiplying both parts of the equation (1) by $\mathbf{A}_0^{-1}$, we obtain

$$X_t = \mathbf{A}X_{t-1} + \Pi\epsilon_t \tag{2}$$

where $\mathbf{A} \equiv \mathbf{A}_0^{-1}\mathbf{A}_1$, $\Pi \equiv \mathbf{A}_0^{-1}\Sigma^{1/2}$, and $\epsilon_t \equiv \Sigma^{-1/2}\mu_t$.
Here ϵ_t is a 7x1 random vector with identity variance-covariance matrix and mean zero.

The paper follows Schmitt-Grohe and Uribe (2018) and assumes that for Kazakhstan, a small emerging economy, the terms of trade shocks and interest spread shocks are exogenously given. Hence, the identification restriction placed on matrix $\mathbf{A}_1$ is that all elements of the first and second rows of the matrix except for the first and second should be zero. The identification scheme that the paper employs for the terms of trade shock assumes that there exists a contemporaneous effect of the terms of trade shocks on the interest rate spread. However, the spread shock has an effect on the terms of trade only after one period. Thus, the relation between terms of trade and spread takes the following form

$$\begin{bmatrix} ToT_t \\ S_t \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} ToT_{t-1} \\ S_{t-1} \end{bmatrix} + \begin{bmatrix} \pi_{11} & 0 \\ \pi_{21} & \pi_{22} \end{bmatrix} \begin{bmatrix} \epsilon_t^1 \\ \epsilon_t^2 \end{bmatrix} \tag{3}$$

Equation 3 represents first two equations of the SVAR model with α_{ij} and π_{ij} denoting the elements (i, j) of the matrices $\mathbf{A}$ and Π, respectively. The innovation to the terms of trade equation, ϵ_t^1, represents the terms of trade shock and ϵ_t^2 represents an interest spread shock. The paper estimates the SVAR model equation by equation via OLS.

Table 2 displays parameter estimates of Equation 3 for Kazakhstani economy and the median values of the same equation estimates in Schmitt-Grohe and Uribe (2018). The estimated autocorrelation coefficient of the terms of trade, α_{11}, is 0.59. The median value of this coefficient in Schmitt-Grohe and Uribe (2018) is 0.55. However, the median absolute deviation constitutes around 21 percent of the median value. Hence, it might be argued that terms of trade autocorrelation considerably differs across countries. The difference occurs in the estimates of α_{12}, the coefficient of the lagged interest spread effect on the terms of trade. The estimated parameter for α_{12} for Kazakhstan is -0.93, while the median value in Schmitt-Grohe and Uribe (2018) is around 0.60. Further, the standard deviation of the innovation term to the terms of trade for Kazakhstan, π_{11}, is 0.04, and the standard deviation of the innovation term to the interest rate spread, π_{22}, is 0.005.

	α_{11}	α_{12}	α_{21}	α_{22}	π_{11}	π_{21}	π_{22}
Kazakhstan	0.59	-0.94	0.03	0.92	0.04	-0.002	0.005
SGU (2018)	0.55	0.60	-0.02	0.52	0.084	-0.0007	0.013

Table 2: Parameter estimates

Figure A.2 in Appendix A presents impulse responses of endogenous variables in the vector X_t to a 5 % increase in the terms of trade for Kazakhstani economy with confidence interval of 66 %. The choice of a 5 % improvement stems from the result of the parameter estimate for the standard deviation of the innovation term to the terms of trade, which is close to 5 %. The effect of the terms of trade improvement on the trade balance is positive, but very small. An increase in the terms of trade by 5% leads to an improvement of the trade balance by only 0.06 %. Thus, the support for the Harberger-Laursen-Metzler (HLM) effect is very weak. On the other hand, Schmitt-Grohe and Uribe (2018) find the evidence for HLM effect for the majority of countries in the sample. Next, an increase in the terms of trade improves the aggregate economic activity in Kazakhstan. GDP and private consumption increase by almost 1% in response to a 5% increase in the terms of trade. Investment increases by more than 1%, but the effect disappears in the next period. Further, the response of investment reaches its minimum and turns negative in the second period when the response of consumption reaches its maximum. The real exchange rate appreciates by 3 % in the next three quarters. In other words, Kazakhstan becomes more expensive vis-a-vis the rest of the world in response to a positive terms of trade shock. The results are robust to using the index for real effective exchange rate, which is measured using trade weights.

In order to determine the importance of terms of trade shocks to business cycle fluctuations for Kazakhstani economy the paper analyzes the share of variance of each endogenous variable explained by these shocks. Table 3 displays the fraction of variance of variables of interest explained by the terms of trade shocks for Kazakhstan and the median values of the same parameters from Schmitt-Grohe and Uribe (2018) analysis. The estimates indicate that the terms of trade shocks account for 12% of the variance in output, 2% of variance in the trade balance, 8% in consumption, 2% in investment and 46% of the variance in the real exchange rate for the period between 2001 and 2018. The results of the estimation are robust to using the Bloomberg commodity index (BCI) instead of the terms of trade. Schmitt-Grohe and Uribe (2018) conclude that terms of trade shocks, on average, explain around 10% of the variance of the aforementioned variables. However, the authors admit that cross-country variation is significant in their analysis.

	ToT	S	TB	GDP	C	I	RER
ToT	0.88	0.09	0.02	0.12	0.08	0.02	0.46
BCI	0.81	0.05	0.08	0.13	0.04	0.01	0.41
SGU (2018)	0.95	0.09	0.11	0.07	0.10	0.09	0.11

Table 3: Variance decomposition of the terms of trade shock

4. The theoretical model

The paper employs the MXN model developed by Schmitt-Grohe and Uribe (2018) with an importable (m), an exportable (x), and a nontradable (n) sectors. The MXN model adds three generalizations to the model, developed by Mendoza (1995). First, employment in exportable and importable sectors endogenously vary over the business cylce. Second, following McIntyre (2003), capital can be accumulated in the nontradable sector as well. Third, because sizable fraction of investment constitutes of nontradable goods, the model assumes that investment goods are not entirely imported. Following Schmitt-Grohe and Uribe (2018), exogenously set parameters in the model are specific to Kazakhstani economy. Further, key structural parameters are estimated within the model. Finally, theoretical counterparts of empirical variables in the model are expressed in a consistent way. The full specification of the model structure is provided in Appendix B.

5. Calibration, estimation, and impulse responses

Most papers on DSGE models for small open economies take parameter values common for all emerging economies. Sometimes due to the unavailability of data and research on small open economies, authors apply general parameter values estimated for advanced economies. Very little research is done on macroeconomic parameter values in DSGE models for Kazakhstani economy. Mukhamediev and Kakizhanova (2014) use time-series data and Bayesian method to calibrate the model parameters. Abilov (2020) employs the Bayesian estimation method to find posterior parameter values. Another research done on the parameters of DSGE models is based on micro-level data. Adilkhanova (2019) estimates most macroeconomic parameters for Kazakhstani economy using rich household- and firm-level data for the period between 2009 and 2018. This paper takes most of the exogenously set parameters from Adilkhanova (2019). Some parameters are determined in the steady state. Moment restrictions on the sectoral and value added share values are obtained from time series data. Following Schmitt-Grohe and Uribe (2018), capital adjustment cost parameters, ϕ_j, and the parameter of the debt elasticity of the country risk premium, ψ, are estimated by minimizing the weighted distance between impulse response functions of the SVAR and theoretical models to the terms of trade and interest rate spread shocks. Adilkhanova (2019) estimates the constant relative risk aversion coefficient, σ, to be 2.71. It is quite larger than the common value for the parameter in business cycle analysis and implies that households in Kazakhstan are more likely to be risk averse. The value for ω_j is 2.28, which implies a Frisch elasticity of labor supply of 0.78. This parameter is the developing country benchmark in Mendoza (1995). We set the same value for all three sectors due to the unavailability of sector-level information. The depreciation level, δ is estimated to be 0.06 per year. The value for $r^* + \bar{s}$ is set to 0.06 per year and reflects the sum of average values of federal funds rate and the spread for the period between 2001 and 2018. The elasticity of substitution between importable and exportable goods, μ_{mx}, and the elasticity of substitution between tradables and nontradables, $\mu_{\tau n}$ are set to 0.54 and 0.76, respectively. Further, according to Adilkhanova (2019), the capital share in the nontradable sector is set to 0.27, while the capital shares in the exportable and importable sectors are set to 0.35. Using Statistics Committee database we calculate the average ratio of value added exports to GDP for Kazakhstan to be around 35 % for the period between 2001 to 2018. Hence, we set $x/(p^m y^m + p^x y^x + p^n y^n) = 0.35$. The share of nontradable sector, $p^n y^n/(p^m y^m + p^x y^x + p^n y^n)$, is equal to 30 %. Unlike Schmitt-Grohe and Uribe (2018), we set the shares of exportable sector to be larger than the share of importable sector, that is we impose $p^x y^x > p^m y^m$. The values for α_{ij} and π_{ij}, for $i, j = 1, 2$, are assigned using the estimates of the SVAR analysis from Section 3. Table A.5 in Appendix A provides the calibration and estimation results of the model parameters.

Figure A.3 in Appendix A depicts impulse responses of key macroeconomic variables to the terms of trade shock in the SVAR and the DSGE models. Overall, it can be argued that the model modestly fits the data. The impulse responses of output, consumption and investment in the theoretical model is similar in magnitude and timing to the one in the SVAR model. The DSGE mode well predicts the appreciation of the real exchange rate. However, it is not able to generate the hump-shaped response of the real exchange rate to the shock. Further, the theoretical model does not find any support for an HLM effect. The trade balance negatively responds to the terms of trade shock in the model. Hence, the DSGE model predictions support the Obstfeld-

Razin-Svensson (ORS) effect, brought by Obstfeld (1982) and Svensson and Razin (1983). The authors state that persistence of the terms of trade plays an important in the determination of the effect of the terms of trade shocks on the trade balance. In other words, the ORS effect argues that if the terms of trade shocks are permanent, the relation between the terms of trade and the trade balance may weaken or even become negative.

To better understand the transmission mechanism of the terms of trade shocks in the DSGE model, we examine impulse responses of the variables that are not included in the SVAR model. Figure A.4 in Appendix A displays impulse responses to a 5 % improvement in the terms of trade implied by the theoretical model. To begin with, in line with the data, the DSGE model predicts the appreciation of the real exchange rate in response to a positive terms of trade shock. Further, the relative price of nontradable goods increases. This response is attributed to the substitution and income effects. First, an increase in the relative price of exportables leads to a substitution of domestic absorption of exportables with nontradable and importable absorption. Second, an increase in the relative price of exportables results in a positive income effect that increases the domestic demand for all types of goods. Hence, expansion in the demand for nontradable goods leads to an increase in price of this type of good.

The second row of the Figure A.4 shows that both exports and imports increase in response to the positive terms of trade shock. Producers of exportable goods increase the supply in response to a price change, while the domestic absorption of exportables decreases. Imports, on the other hand, increase because the importable sector cuts back supply, while the domestic absorption for importable goods increases. Overall, the trade balance slightly deteriorates.

Total output expands in response to the improvement in the terms of trade. This effect is mostly driven by the output expansion in exportable and nontradable sectors. Output in importable sector, as mentioned above, contracts in response to the positive terms of trade shock. Investment in exportable and nontradable sectors imitates the dynamics of sectoral production.

6. Explained variance

What does the DSGE model tell us about the variance of macroeconomic aggregates explained by the terms of trade shocks? Figure A.5 in Appendix A depicts the share of variance of output, consumption, investment, the real exchange rate, and the trade balance attributed to the terms of trade shocks according to the SVAR and DSGE models. Predictions of the theoretical model are fairly consistent with the predictions of the empirical model. The considerable divergence occurs in the prediction of the variance of the real exchange rate. The theoretical model attributes much less variance in the real exchange rate to the terms of trade than the empirical model. This might be the result of the definition of the real exchange rate in the DSGE model. Schmitt-Grohe and Uribe (2018) obtain same results on the real exchange rate. Table A.4 provides numerical values of the two models' predictions on the variance of macroeconomic aggregates. The three sectoral DSGE model also attributes 12% of the variance in the output to the terms of trade shocks. In addition, the shares of the variance of consumption, investment and the trade balance are in line with the data as well.

7. Conclusion

To conclude, an empirical SVAR estimation shows that the terms of trade shocks play a modest role in generating business cycle fluctuations in Kazakhstan. The empirical model based on a quarterly data from 2001 until 2018 reveal that the terms of trade shocks account for 12 % of output variance. Three sectoral DSGE model modestly fits the empirical results of the SVAR model. The theoretical model simulates well the magnitude and the behavior of output, consumption and investment in response to a positive terms of trade shock. In addition, the three sectoral DSGE model lines up with the SVAR model in explaining the variance of four endogenous variables except the real exchange rate. The magnitude of the response and the variance of the real exchange rate is not well captured in the model.

Acknowledgment

The author would like to express his gratitude to Professor Xavier Mateos-Planas (Queen Mary University of London) for the guidance throughout the research and valuable comments.

Declaration of interest statement

The author declares that he has no competing interests.

References

N. Abilov. An Estimated Bayesian DSGE model for Kazakhstan. *Asian Journal of Economic Modeling*, 8(1):34–54, 2020.

Z. Adilkhanova. Microlevel Analyses of DSGE Model Parameters: Evidence from Kazakhstan. *NAC Anaytica Working Papers No. 2*, 2019.

O. Akinci. Global financial conditions, country spreads and macroeconomic fluctuations in emerging countries. *Journal of International Economics*, 91: 358–371, 2013.

T. Drechsel and S. Tenreyro. Commodity booms and busts in emerging economies. *Journal of International Economics*, 112:200—218, 2018.

K. H. McIntyre. Can non-traded goods solve the 'comovement problem'. *Journal of Macroeconomics*, 25:169–196, 2003.

E. Mendoza. The terms of trade, the real exchange rate, and economic fluctuations. *International Economic Review*, 36(1):101–137, 1995.

B. Mukhamediev and T. Kakizhanova. Modelling of oil revenues impact to the dynamics of main macroeconomic indicators of Kazakhstan. *KazNU Bulletin. Economics series*, 101(1), 2014.

M. Obstfeld. Aggregate spending and the terms of trade: Is there a Laursen-Metzler effect? *Quarterly Journal of Economics*, 97:251–270, 1982.

S. Schmitt-Grohe and M. Uribe. Closing small open economy models. *Journal of International Economics*, 61:163–185, 2003.

S. Schmitt-Grohe and M. Uribe. How important are terms-of-trade shocks? *International Economic Review*, 59(1):85–111, 2018.

L. E. O. Svensson and A. Razin. The terms of trade and the current account: The Harberger-Laursen-Metzler effect. *Journal of Political Economy*, 91:97–125, 1983.

Appendix A.

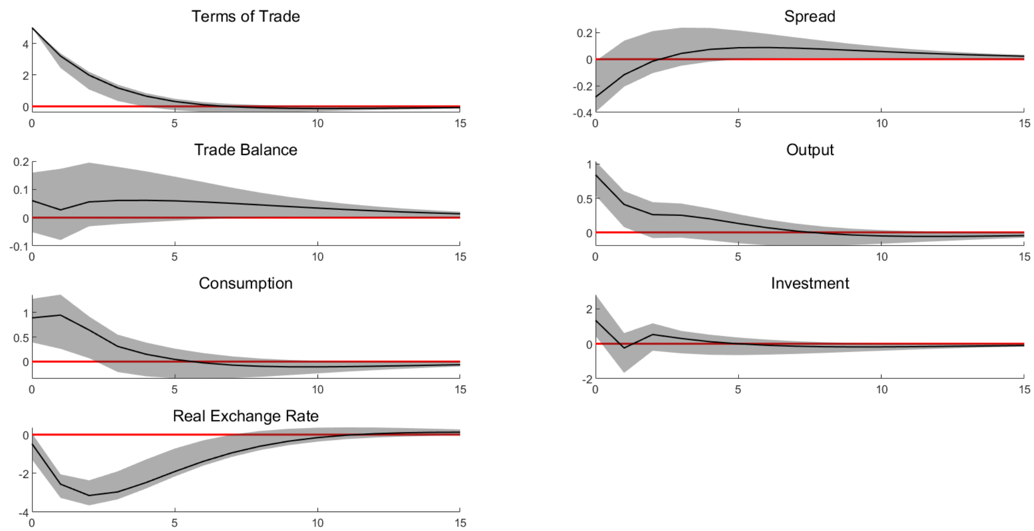

Figure A.2: Impulse responses to a 5% increase in the terms of trade

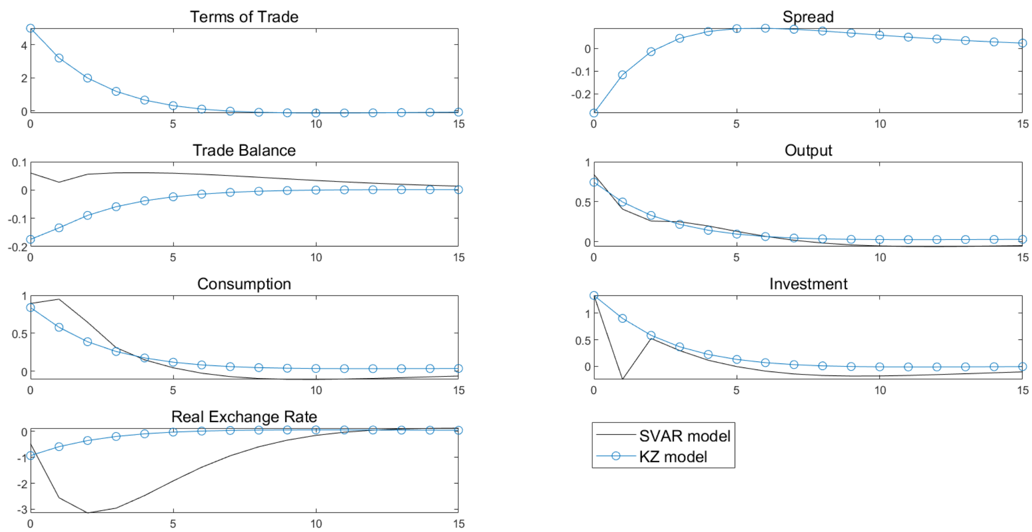

Figure A.3: Impulse responses to a 5% increase in the terms of trade: data versus model

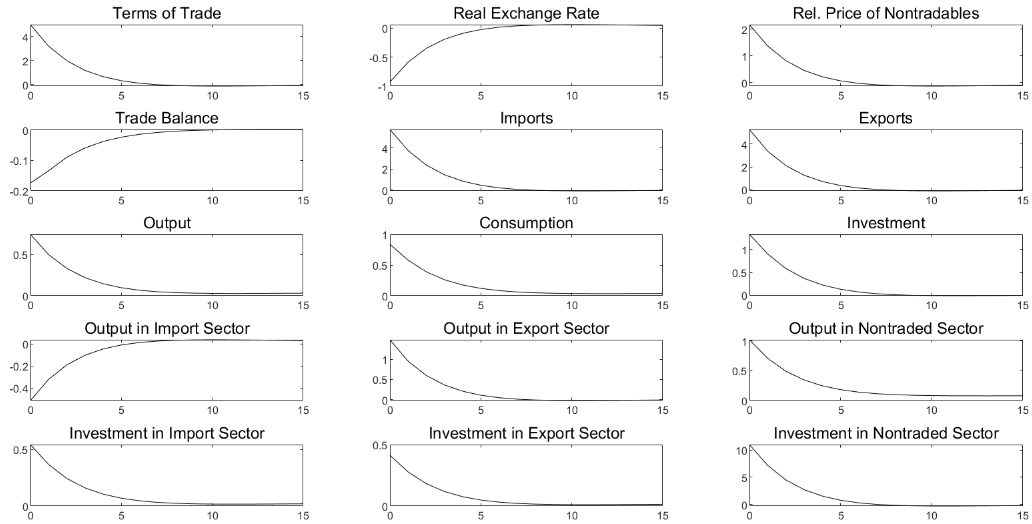

Figure A.4: Impulse responses to a 5% increase in the terms of trade

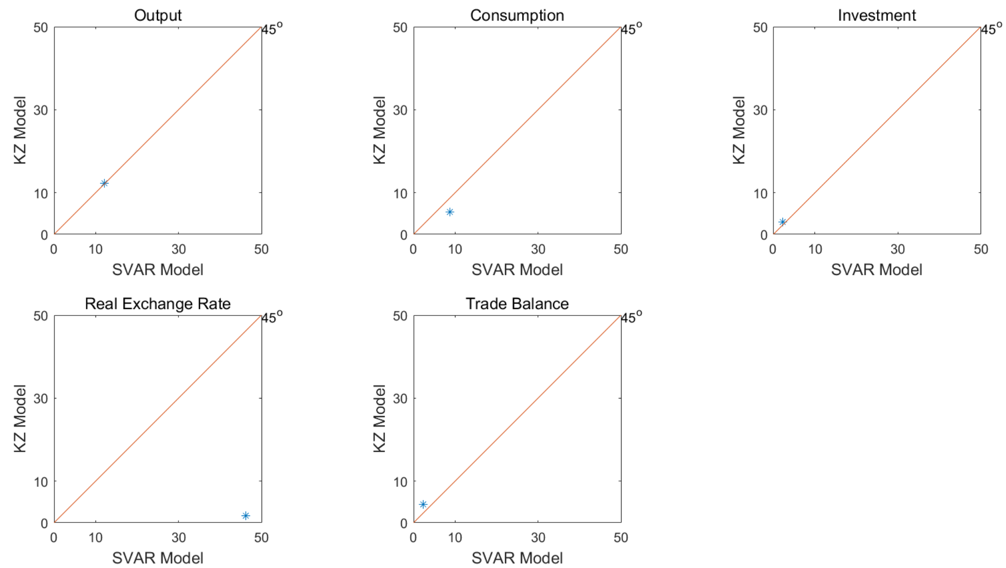

Figure A.5: Variance of output, consumption, investment, the real exchange rate, and the trade balance explained by the terms of trade shocks according to the SVAR and DSGE models

	Trade Balance	Output	Consumption	Investment	RER
SVAR	2.25	12.16	8.71	2.26	46.29
DSGE	4.44	12.31	5.36	3.01	1.72

Table A.4: Shares of variance explained by the terms of trade shocks

Calibrated structural parameters										
σ	δ	$r^* + \overline{s}$	α_m, α_x	α_n	ω_j	μ_{mx}	$\mu_{\tau n}$	$\overline{tot}$	A_m, A_n	β
2.71	0.06	0.06	0.35	0.27	2.28	0.54	0.76	1	1	$\frac{1}{1+r^*+\overline{s}}$

Moment restrictions			
s_n	s_x	s_{tb}	$\frac{p^m y^m}{p^x y^x}$
0.3	0.35	0.001	0.75

Determined structural parameters				
χ_m	χ_τ	$\overline{d}$	A^x	β
0.9914	0.7580	0.3883	1.1107	0.9852

Estimated parameters										
ϕ_m	ϕ_x	ϕ_n	ψ	α_{11}	α_{12}	α_{21}	α_{22}	π_{11}	π_{21}	π_{22}
77.09	78.77	4.81	34.92	0.59	-0.94	0.03	0.92	0.04	-0.002	0.005

Table A.5: Parameters of the model

Appendix B.

The Model

Households

There is a representative household with the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t^m, h_t^x, h_t^n), \tag{B.1}$$

where c_t denotes consumption and h_t^j denotes number of hours worked in sector j with $j = m, x, n$. The representative household's budget constraint is given by

$$\begin{aligned} c_t + i_t^m + i_t^x + i_t^n + \Phi_m(k_{t+1}^m - k_t^m) + \Phi_x(k_{t+1}^x - k_t^x) + \Phi_n(k_{t+1}^n - k_t^n) + p_t^\tau d_t \\ = \frac{p_t^\tau d_{t+1}}{1 + r_t} + w_t^m h_t^m + w_t^x h_t^x + w_t^n h_t^n + u_t^m k_t^m + u_t^x k_t^x + u_t^n k_t^n, \end{aligned}$$

where $i_t^j, k_t^j, w_t^j, u_t^j$ denote investment, capital stock, real wage, and rental rate of capital in sector j, respectively. $\Phi_j(\cdot)$ are nonnegative and convex functions that represent capital adjustment costs and satisfy $\Phi_j(\cdot) = \Phi_j^{'}(\cdot) = 0$. p_t^τ is the relative price of tradable composite goods in terms of final goods, d_t is the stock of debt held by the household in period t. Debt is expressed in the units of tradable goods and the variable r_t denotes one period interest rate on debt. The other remaining variables are all expressed in units of final goods. The law of motion of the capital stock is given by

$$k_{t+1}^m = (1 - \delta) k_t^m + i_t^m, \tag{B.2}$$

$$k_{t+1}^x = (1 - \delta) k_t^x + i_t^x, \tag{B.3}$$

$$k_{t+1}^n = (1 - \delta) k_t^n + i_t^n, \tag{B.4}$$

By replacing i_t^j in the household's budget constraint with $k_{t+1}^j - (1-\delta)k_t^j$ and maximizing the utility function subject to the budget constraint with respect to consumption, hours worked, debt, and capital stock we obtain the following optimality conditions

$$U_1(c_t, h_t^m, h_t^x, h_t^n) = \lambda_t \tag{B.5}$$

$$-U_2(c_t, h_t^m, h_t^x, h_t^n) = \lambda_t w_t^m \tag{B.6}$$

$$-U_3(c_t, h_t^m, h_t^x, h_t^n) = \lambda_t w_t^x \tag{B.7}$$

$$-U_4(c_t, h_t^m, h_t^x, h_t^n) = \lambda_t w_t^n \tag{B.8}$$

$$\lambda_t p_t^\tau = \beta(1+r_t)E_t \lambda_{t+1} p_{t+1}^\tau \tag{B.9}$$

$$\lambda_t[1 + \Phi_m'(k_{t+1}^m - k_t^m)] = \beta E \lambda_{t+1}[u_{t+1}^m + (1-\delta) + \Phi_m'(k_{t+2}^m - k_{t+1}^m)] \tag{B.10}$$

$$\lambda_t[1 + \Phi_x'(k_{t+1}^x - k_t^x)] = \beta E \lambda_{t+1}[u_{t+1}^x + (1-\delta) + \Phi_x'(k_{t+2}^x - k_{t+1}^x)] \tag{B.11}$$

$$\lambda_t[1 + \Phi_n'(k_{t+1}^n - k_t^n)] = \beta E \lambda_{t+1}[u_{t+1}^n + (1-\delta) + \Phi_n'(k_{t+2}^n - k_{t+1}^n)] \tag{B.12}$$

The above expressions show that rates of return on capital are equalized in the steady state, whereas wage differences are still present in the steady state.

Final goods producers

Firms producing final goods use the aggregator function $B(a_t^\tau, a_t^n)$, where a_t^τ represents domestic absorption of tradable composite goods and a_t^n denotes domestic absorption of nontradable goods. The market, where final goods are produced is assumed to be competitive. Profits of final goods producers are given by

$$B(a_t^\tau, a_t^n) - p_t^\tau a_t^\tau - p_t^n a_t^n,$$

where p_t^τ and p_t^n are relative prices of tradable and nontradable goods in terms of final goods. Then, profit maximization conditions for the firm are

$$B_1(a_t^\tau, a_t^n) = p_t^\tau \tag{B.13}$$

$$B_2(a_t^\tau, a_t^n) = p_t^n. \tag{B.14}$$

Equations (B.13) and (B.14) define domestic demand functions for tradable and nontradable goods.

Tradable goods producers

Tradable goods are composite of importable and exportable goods

$$a_t^\tau = A(a_t^m, a_t^x), \tag{B.15}$$

where $A(\cdot, \cdot)$ is the aggregator function for a_t^m, the domestic absorption of importable goods, and a_t^x, the domestic absorption of exportable goods. Profits are defined as

$$p_t^\tau A(a_t^m, a_t^x) - p_t^m a_t^m - p_t^x a_t^x,$$

where p_t^m and p_t^x denote relative price of importable and exportable goods in terms of final goods, respectively. Firms in the tradable goods sector are assumed to behave competitively, thus profit maximization conditions are given by

$$p_t^\tau A_1(a_t^m, a_t^x) = p_t^m \tag{B.16}$$

$$p_t^\tau A_2(a_t^m, a_t^x) = p_t^x. \tag{B.17}$$

The above two equations represent domestic demand for importable and exportable goods.

Importable, exportable, and nontradable goods producers

Firms producing importable, exportable, and nontradable goods use capital and labor as inputs of production via the technologies

$$y_t^m = A^m F^m(k_t^m, h_t^m), \tag{B.18}$$

$$y_t^x = A^x F^x(k_t^x, h_t^x), \tag{B.19}$$

$$y_t^n = A^n F^n(k_t^n, h_t^n), \tag{B.20}$$

where y_t^j denotes output in sector j, A^j denotes total factor productivity in sector j, and $F^j(\cdot,\cdot)$ are increasing, concave production functions homogeneous of degree 1. Profits of firms operating in these sectors are given by

$$p_t^j A^j F^j(k_t^j, h_t^j) - w_t^j h_t^j - u_t^j k_t^j,$$

Under the assumption of competitive behavior of firms producing importable, exportable, and nontradable goods the profit maximization conditions are given by

$$p_t^m A^m F_1^m(k_t^m, h_t^m) = u_t^m \tag{B.21}$$

$$p_t^x A^x F_1^x(k_t^x, h_t^x) = u_t^x \tag{B.22}$$

$$p_t^n A^n F_1^n(k_t^n, h_t^n) = u_t^n \tag{B.23}$$

$$p_t^m A^m F_2^m(k_t^m, h_t^m) = w_t^m \tag{B.24}$$

$$p_t^x A^x F_2^x(k_t^x, h_t^x) = w_t^x \tag{B.25}$$

$$p_t^n A^n F_2^n(k_t^n, h_t^n) = w_t^n. \tag{B.26}$$

Equations (B.21) - (B.23) represent the sectoral demand for capital, while equations (B.24) - (B.26) represent demand for labor.

Equilibrium

Equilibrium condition requires the equality of demand for final goods and supply of these goods:

$$c_t + i_t^m + i_t^x + i_t^n + \Phi_m(k_{t+1}^m - k_t^m) + \Phi_x(k_{t+1}^x - k_t^x) + \Phi_n(k_{t+1}^n - k_t^n) = B(a_t^\tau, a_t^n). \tag{B.27}$$

Nontradable goods are wholly consumed domestically. Hence,

$$a_t^n = y_t^n. \tag{B.28}$$

Imports and exports, denoted m_t and x_t, respectively, are defined as the difference between domestic demand for these types of goods and output in the corresponding sector:

$$m_t = p_t^m(a_t^m - y_t^m) \tag{B.29}$$

$$x_t = p_t^x(y_t^x - a_t^x). \tag{B.30}$$

Imports and exports are expressed in units of final goods, while the domestic absorption and output are expressed in units of corresponding goods. Thus, the right-hand side of equations (B.29) and (B.30) includes the price of importables and exportables, respectively.

Integrating the equilibrium condition, equations for imports and exports, the household's budget constraint, and the expressions for profits of firms, results in the following economy-wide resource constraint:

$$p_t^\tau d = \frac{p_t^\tau d_{t+1}}{1+r_t} + x_t - m_t \tag{B.31}$$

Standard small open economy models suffer from nonstationarity in equilibrium dynamics. This results in computational difficulties because computational techniques require stationary process. In small open economy models, households have only access to a risk-free bond with rate that is exogenously determined. Hence, initial debt position affects the steady state of the model. Therefore, to induce stationarity in small open economy models Schmitt-Grohe and Uribe (2003) evaluate five alternative modifications to the model and find that quantitative predictions of macroeconomic variables of all modifications conditional on same calibration are similar. Hence, the model introduces debt elastic interest rate faced by domestic households. In particular, the interest rate, r_t, is given by

$$r_t = r^* + s_t + p(d_{t+1}), \tag{B.32}$$

where r^* is the risk-free world interest rate, s_t and $p(d_{t+1})$ are global and domestic components of interest rate spread, respectively.

Terms of trade is defined as a relative price of exports in terms of imports.

Hence, we have that

$$tot_t = \frac{p_t^x}{p_t^m}. \tag{B.33}$$

In line with Section 3, it is assumed that the evolution of the terms of trade and the global component of interest rate spread are exogenously given.

The real exchange rate is defined in the following way:

$$q_t = \frac{\varepsilon_t P_t^*}{P_t},$$

where q_t denotes the real exchange rate, ε_t is the nominal exchange rate, and P_t^* and P_t are prices of consumption in the foreign and home countries, respectively. Next, dividing the numerator and the denominator of the real exchange rate by the domestic price of tradable goods gives $q_t = (\varepsilon_t P_t^*/P_t^\tau)/(P_t/P_t^\tau)$. If importable and exportable goods aggregator technology is same accross countries and the law of one price for these types of goods holds, then the law of one price must hold for tradable goods as well. The law of one price for tradable goods states that $\varepsilon_t P_t^{\tau *} = P_t^\tau$. Hence, the real exchange rate can be defined as

$$q_t = \frac{P_t^*/P_t^{\tau *}}{P_t/P_t^\tau}.$$

Further, under the assumption that terms of trade shocks that are relevant for a small open economy have no impact on the relative price of tradable goods in foreign country, we obtain that the numerator of the above real exchange rate equation is constant. Then, normalizing the numerator of the real exchange rate to unity and noting that P_t^τ/P_t is the domestic relative price of tradable goods, we obtain

$$q_t = p_t^\tau. \tag{B.34}$$

Equation (B.34) states that real exchange rate can be defined as a relative price of tradable goods in terms of final consumption goods.

Nominal vs. Real

In the model, consumption, c_t, is expressed in units of final consumption goods. GDP, investment, and the trade balance are also expressed in units of final goods and are given by

$$y_t = p_t^m y^m + p_t^x y^x + p_t^n y^n, \tag{B.35}$$

$$i_t = i_t^m + i_t^x + i_t^n, \tag{B.36}$$

$$tb_t = x_t - m_t. \tag{B.37}$$

The data employed in the empirical model, however, are measured at constant prices. Therefore, for a meaningful comparison of the empirical and theoretical models there is a need to express the observable variables in the same units. In other words, the variables of interest in the theoretical model should be expressed at constant prices. GDP deflator, which is given by

$$\frac{P_t^m y_t^m + P_t^x y_t^x + P_t^n y_t^n}{P_0^m y_t^m + P_0^x y_t^x + P_0^n y_t^n},$$

where the numerator denotes GDP at current prices and the denominator denotes GDP at constant prices, can be used to obtain real values of variables in the theoretical model. Then, real GDP is given by

$$P_0^m y_t^m + P_0^x y_t^x + P_0^n y_t^n.$$

If we set the nominal price of final consumption goods in the base year equal to one, then nominal sectoral base year prices, P_0^j, are equal to the relative sectoral prices in terms of final consumption goods, p_0^j in the base year. Then, we can rewrite the real output

$$p_0^m y_t^m + p_0^x y_t^x + p_0^n y_t^n.$$

Finally, we assume that the economy in the base year is in the deterministic steady state. In other words, we can remove the subscript zero from relative sectoral prices so that $p_0^j = p^j$. Hence, the real output in the model, denoted by y_t^0, is given by

$$y_t^0 = p^m y_t^m + p^x y_t^x + p^n y_t^n. \tag{B.38}$$

Similarly, using the same steps as with the determination of the real output in the model we can derive the theoretical counterparts of observed real consumption, investment, and the trade balance. These are given by

$$c_t^0 = \frac{c_t}{y_t} y_t^0, \tag{B.39}$$

$$i_t^0 = \frac{i_t}{y_t} y_t^0, \tag{B.40}$$

$$tb_t^0 = \frac{tb_t}{y_t} y_t^0. \tag{B.41}$$

Functional forms

The utility function displays constant relative risk aversion and takes the form

$$U(c, h^m, h^x, h^n) = \frac{\left[c - \frac{(h^m)^{\omega_m}}{\omega_m} - \frac{(h^x)^{\omega_x}}{\omega_x} - \frac{(h^n)^{\omega_n}}{\omega_n}\right]^{1-\sigma} - 1}{1-\sigma},$$

where σ, $\omega_j > 0$, and denote the relative risk aversion and sectoral elastisity of labor supply, respectively.

Firms producing importable, exportable, and nontradable goods are assumed to use Cobb-Douglas production function,

$$F^m(k^m, h^m) = (k^m)^{\alpha_m} (h^m)^{1-\alpha_m},$$

$$F^x(k^x, h^x) = (k^x)^{\alpha_x}(h^x)^{1-\alpha_x},$$

$$F^n(k^n, h^n) = (k^n)^{\alpha_n}(h^n)^{1-\alpha_n},$$

where α_j denotes the sectoral share of capital. Next, tradable and final goods are bundled using the Armington aggregators with constant elasticity of substitution,

$$A(a_t^m, a_t^x) = \left[\chi_m(a_t^m)^{\frac{\mu_{mx}-1}{\mu_{mx}}} + (1-\chi_m)(a_t^x)^{\frac{\mu_{mx}-1}{\mu_{mx}}}\right]^{\frac{\mu_{mx}}{\mu_{mx}-1}},$$

$$B(a_t^\tau, a_t^n) = \left[\chi_\tau(a_t^\tau)^{\frac{\mu_{\tau n}-1}{\mu_{\tau n}}} + (1-\chi_\tau)(a_t^n)^{\frac{\mu_{\tau n}-1}{\mu_{\tau n}}}\right]^{\frac{\mu_{\tau n}}{\mu_{\tau n}-1}},$$

where χ_m and χ_τ are shares of importable and tradable goods in the production, and μ_{mx}, $\mu_{\tau n}$ are elastisities of substitution between importables and exportables, and between tradeables and nontradables, respectively. Interest rate premium and capital adjustment costs are given by

$$P(d) = \psi\left(e^{d-\overline{d}} - 1\right),$$

$$\Phi_j(\Delta k) = \frac{\phi_j}{2}(\Delta k)^2.$$